Nick Hunter

In[1]:=

Derivation

Generalized Coordinates

In[2]:=
```
x[ϕ_, θ_] = L2 * Cos[θ[t]] + R * Sin[ϕ[t]];
y[ϕ_, θ_] = (L2 + R) + (L2 * Sin[θ[t]] - R * Cos[ϕ[t]]);
x'[ϕ_, θ_] = ∂t x[ϕ, θ];
y'[ϕ_, θ_] = ∂t y[ϕ, θ];
X[ϕ_, θ_] = -L1 * Cos[θ[t]];
Y[ϕ_, θ_] = (L2 + R) - L1 * Sin[θ[t]];
X'[ϕ_, θ_] = ∂t X[ϕ, θ];
Y'[ϕ_, θ_] = ∂t Y[ϕ, θ];
```

Lagrangian

In[10]:=
```
T[ϕ_, θ_] =
  FullSimplify[1/2 (M1 (X'[ϕ, θ]^2 + Y'[ϕ, θ]^2) + M2 (x'[ϕ, θ]^2 + y'[ϕ, θ]^2))];
U[ϕ_, θ_] = FullSimplify[g (M1 * Y[ϕ, θ] + M2 * y[ϕ, θ])];
L[ϕ_, θ_] = FullSimplify[T[ϕ, θ] - U[ϕ, θ]];
```

Equations Of Motion

In[13]:=
```
Needs["VariationalMethods`"];
θ''[t_] = FullSimplify[Solve[EulerEquations[L[ϕ, θ], θ[t], t], θ''[t]][[1, 1, 2]]];
ϕ''[t_] = FullSimplify[Solve[EulerEquations[L[ϕ, θ], ϕ[t], t], ϕ''[t]][[1, 1, 2]]];
```

In[16]:=
```
θA[ϕ_, θ_] = FullSimplify[θ''[t]]
ϕA[ϕ_, θ_] = FullSimplify[ϕ''[t]]
```

Out[16]=

$$\frac{g\,\mathrm{Cos}[\theta[t]]\,L_1 M_1 - \mathrm{Cos}[\theta[t]-\phi[t]]\,L_2 M_2\left(g\,\mathrm{Cos}[\phi[t]] - \mathrm{Sin}[\theta[t]-\phi[t]]\,L_2\,\theta'[t]^2 + R\,\phi'[t]^2\right)}{L_1^2 M_1 + \mathrm{Cos}[\theta[t]-\phi[t]]^2 L_2^2 M_2}$$

Out[17]=

$$\frac{1}{R\left(L_1^2 M_1 + \mathrm{Cos}[\theta[t]-\phi[t]]^2 L_2^2 M_2\right)}\Big(g\,\mathrm{Cos}[\theta[t]]\,\mathrm{Sin}[\theta[t]-\phi[t]]\,L_1 L_2 M_1 + L_1^2 M_1\left(-g\,\mathrm{Sin}[\phi[t]] + \mathrm{Cos}[\theta[t]-\phi[t]]\,L_2\,\theta'[t]^2\right) + \mathrm{Cos}[\theta[t]-\phi[t]]\,L_2^2 M_2\left(-g\,\mathrm{Sin}[\theta[t]] + L_2\,\theta'[t]^2 - R\,\mathrm{Sin}[\theta[t]-\phi[t]]\,\phi'[t]^2\right)\Big)$$

Program

Constants

In[18]:=
```
M1 = 4000; L1 = 3; M2 = 50; R = 6; L2 = 25; g = 9.8;
```

Starting Conditions

In[19]:=
```
ϕ[t] = 0; θ[t] = -Pi / 3; θ′[t] = ϕ′[t] = 0; T = FallTime = FinalX = 0;
dt = .01;
```

In[21]:=
```
Data = Reap[
    FallTime = (y'[ϕ, θ] + Sqrt[2 * g * y[ϕ, θ] + (y'[ϕ, θ])^2]) / g;
    FinalX = x[ϕ, θ] + x'[ϕ, θ] * FallTime;
    Sow[{θ[t], θ′[t], θA[ϕ, θ], ϕ[t], ϕ′[t], ϕA[ϕ, θ], {X[ϕ, θ], Y[ϕ, θ]},
      {x[ϕ, θ], y[ϕ, θ]}, {T, FinalX}, {x'[ϕ, θ], y'[ϕ, θ]}}];
    While[ T < 10,
     ϕ′[t] += ϕA[ϕ, θ] * dt;
     θ′[t] += θA[ϕ, θ] * dt;
     ϕ[t] += ϕ′[t] * dt;
     θ[t] += θ′[t] * dt;
     T += dt;
     FallTime = (y'[ϕ, θ] + Sqrt[2 * g * y[ϕ, θ] + (y'[ϕ, θ])^2]) / g;
     FinalX = x[ϕ, θ] + x'[ϕ, θ] * FallTime;
     Sow[{θ[t], θ′[t], θA[ϕ, θ], ϕ[t], ϕ′[t], ϕA[ϕ, θ], {X[ϕ, θ], Y[ϕ, θ]},
       {x[ϕ, θ], y[ϕ, θ]}, {T, FinalX}, {x'[ϕ, θ], y'[ϕ, θ]}}];
    ]];
```

Analysis

Maximum Trajectory

In[22]:=
```
Min[Data[[2, 1, All, 9, 2]]]
```

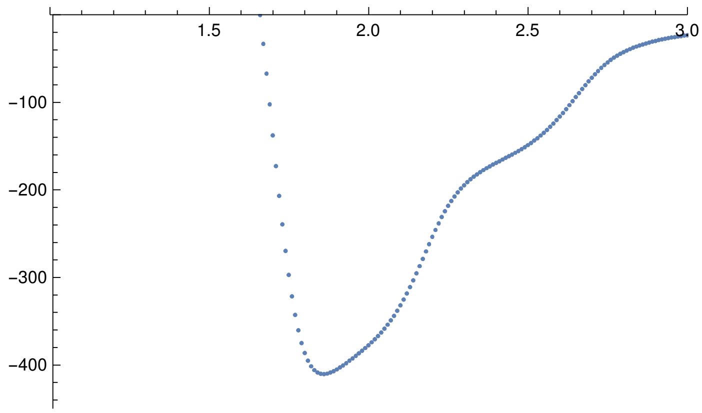
Out[22]= -410.475

Release Time

In[23]:=
```
Data[[2, 1,
  Position[Data[[2, 1, All, 9, 2]], Min[Data[[2, 1, All, 9, 2]]], {1}][[1, 1]], 9, 1]]
```
Out[23]= 1.86

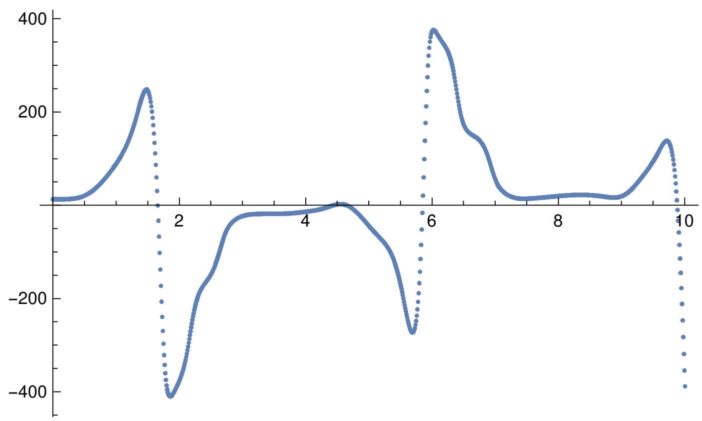
Trajectory Plot At Maximum

In[24]:=
```
ListPlot[Data[[2, 1, All, 9]], PlotRange → {{1, 3}, {-450, 0}}]
```
Out[24]=



Trajectory Plot

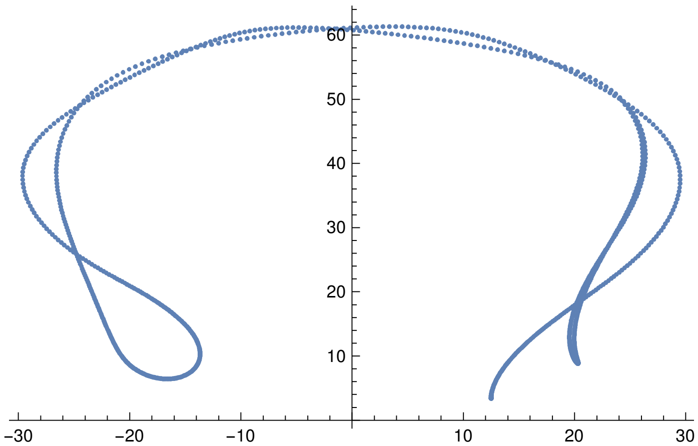
In[25]:=
```
ListPlot[Data[[2, 1, All, 9]], PlotRange → All]
```
Out[25]=



Position of Small Mass

In[26]:=
```
ListPlot[Data[[2, 1, All, 8]], PlotRange → All]
```

Out[26]=



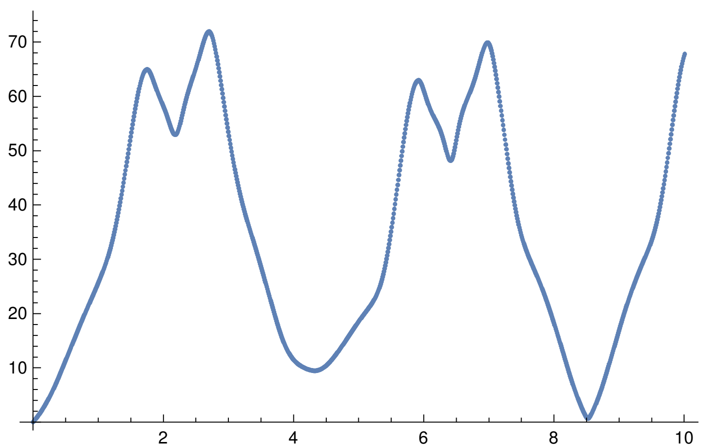
In[27]:=

In[28]:=

Speed (Magnitude(Sqrt[V_x^2 + V_y^2]) Of Small Mass

In[29]:=
```
ListPlot[Table[
  {Data[[2, 1, i, 9, 1]], Sqrt[Data[[2, 1, i, 10, 1]]^2 + Data[[2, 1, i, 10, 2]]^2]},
  {i, 1, Length[Data[[2, 1]]]}], PlotRange → All]
```

Out[29]=



Fun Manipulate Function

In[30]:=
```
Data2 = Table[{Data[[2, 1, i, 7, 1]], Data[[2, 1, i, 7, 2]]},
    {i, 1, Length[Data[[2, 1, All, 2]]]}];
Data3 = Table[{Data[[2, 1, i, 8, 1]], Data[[2, 1, i, 8, 2]]},
    {i, 1, Length[Data[[2, 1, All, 2]]]}];
```

In[32]:=
```
Manipulate[
 Show[ListPlot[Data3, PlotRange → {{-70, 70}, {-70, 70}}, PlotStyle → {White}],
  Graphics[Circle[{Data3[[u, 1]], Data3[[u, 2]]}, 2]],
  Graphics[Circle[{Data2[[u, 1]], Data2[[u, 2]]}, 5]],
  Graphics[Line[{{Data2[[u, 1]], Data2[[u, 2]]},
     {L2 * Cos[Data[[2, 1, u, 1]]], L2 + R + L2 Sin[Data[[2, 1, u, 1]]]}}]],
  Graphics[Line[{{Data3[[u, 1]], Data3[[u, 2]]}, {L2 * Cos[Data[[2, 1, u, 1]]],
      L2 + R + L2 Sin[Data[[2, 1, u, 1]]]}}]]], {u, 1, Length[Data2], 1}]
```

Out[32]=